# Syllabus based on NEP - 2020

for

# M.Sc. Mathematics (Two Year Programme) DOON UNIVERSITY



# DEPARTMENT OF MATHEMATICS, SCHOOL OF PHYSICAL SCIENCES, DOON UNIVERSITY, DEHRADUN-248001, UTTARAKHAND

### 1. Introduction to Postgraduate Degree course in Mathematics

Mathematics is the study of quantity, structure, space and change, focusing on problem solving, with wider scope of application in science, engineering, technology, social sciences etc. The key core areas of study in Mathematics include Algebra, Complex Analysis, Differential Equations, Operations Research, Functional Analysis, Topology, Integral Equations, etc. The Master Degree M.Sc. Mathematics is awarded to the students on the basis of knowledge, understanding, skills, attitudes, values and academic achievements expected to be acquired by learners at the end of the Programme. Learning outcomes of Mathematics are aimed at facilitating the learners to acquire these attributes, keeping in view of their preferences and aspirations for gaining knowledge of Mathematics. Master degree in Mathematics is the culmination of indepth knowledge of algebra, Real analysis, geometry, differential equations and several other branches of Mathematics. This also leads to study of related areas like Computer science, Financial Mathematics. Mathematical Statistics and many more. Thus, this programme helps learners in building a solid foundation for higher studies in Mathematics. The skills and knowledge gained have intrinsic aesthetics leading to proficiency in analytical reasoning. This can be utilised in Mathematical modelling and solving real life problems. Students completing this programme will be able to present Mathematics clearly and precisely, make abstract ideas precise by formulating them in the language of Mathematics, describe Mathematical ideas from multiple perspectives and explain fundamental concepts of Mathematics to non-Mathematicians. Completion of this programme will also enable the learners to join teaching profession, research, enhance their employability for government jobs, jobs in banking, insurance and investment sectors, data analyst jobs and jobs in various other public and private enterprises.

## 2. Courses of Study:

Courses of the study indicate pursuance of study in a particular discipline. Every discipline for master programme shall offer categories of two courses of study, viz. Discipline Specific Core (DSC) courses, Discipline Specific Electives (DSEs) from the respective pool of courses offered by the University.

- (a) Discipline Specific Core (DSC): Discipline Specific Core is a course of study, which should be pursued by a student as a mandatory requirement of his/ her programme of study. In Master of Science Mathematics programme, DSCs are the core credit courses of Mathematics which will be appropriately graded and arranged across the semesters of study, being undertaken by the student, with multiple exit options as per NEP 2020.
- **(b) Discipline Specific Elective (DSE):** The Discipline Specific Electives (DSEs) are a pool of credit courses of Mathematics from which a student will choose to study based on his/her interest.

### 3. Programme Duration and Exit Options:

The minimum credit to be earned by a student per semester is 22 credits. The mandatory number of credits which have to be secured for the purpose of award of Undergraduate Certificate/ Undergraduate Diploma/Appropriate Bachelor's Degree in Mathematics are listed in the following Table 1.

	Table 1: <b>Qual</b>	ification	Type and	d Credit i	Requirements
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S. No.	Type of Award	Stage of Exit	Mandatory Credits to be Secured for the Award
1.	PG Diploma in Mathematics	After successful completion of	44
		Semester II	
2.	Degree of M.Sc in	After successful completion of	88
	Mathematics	Semester IV	

## 4. Programme Objectives:

The postgraduate degree course in Mathematics aims to provide:

a) Deep knowledge of pure, applied and computational mathematical concepts, including, key concepts, principles, theorems, and computer programming through MATLAB, MATHEMATICA,

- and PYTHON.
- b) an introduction to the formulation and solution of practical issues utilising various computational and mathematical tools and methodologies,
- c) ensure that students have the necessary knowledge and abilities to pursue further, more in-depth study in mathematics and related fields,
- d) to deliver high-quality education by implementing projects, participatory learning, and the newest software tools in teaching and learning processes,
- e) to foster in students the ability to think creatively, collaborate with others, and uphold moral principles in order to fulfil societal expectations,
- f) to equip students with the skills they need for successful careers in academia, business, and research,
- g) enough subject-matter expertise to help students prepare for a variety of competitive exams, including the GATE, GRE, UGC-CSIR, NET/JRF, and Civil Services Exams, among others.

## 5. **Programme Outcomes:**

The learning outcomes of the undergraduate degree course in Mathematics are as follows:

- a) Recognize the nature of abstract mathematics and thoroughly investigate the ideas.
- b) Develop mathematical models of real-world issues and derive conclusions by determining the best solutions.
- c) Apply mathematics to theoretical and practical issues by way of critical comprehension, analysis, and synthesis.
- d) Pursue research in challenging areas of pure, applied and computational mathematics.
- e) They will be able to carry out independent study in specific areas of mathematics, instruct mathematics or subjects with a lot of mathematics at the high school, college and degree levels, and work in the industry where mathematics is applied.
- f) Understand, produce, and design documentation relevant to mathematical research and literature, as well as provide persuasive presentations.
- g) Qualify national level tests like NET/GATE in Mathematics etc.

#### 6. Programme Structure:

The detailed framework of postrgraduate degree programme in Mathematics is provided in the following Table-2.

Semester	DSC	DSE	SEC/Project/ Dissertation	Total (credits)
1	(DSC)	(DSE)2x4=8	Seminar/project (1x2=2)	22
	3x4=12	or		
	or	(DSE)3x4=12		
	(DSC) 2x4=8			
2	(DSC)	(DSE)2x4=8	Seminar/project (1x2=2)	22
	3x4=12	or		
	or	(DSE)3x4=12		
	(DSC) 2x4=8			
3	(DSC)2x4=8	(DSE)	Seminar/project (1x2=2)	22
	, ,	3x4=12		
	Or	Or	Or	
			Seminar (1x2=2)	
			Dissertation/Academic	
			Project/Entrepreneurship (1x20=20)	
4	(DSC)2x4=8	(DSE)	Seminar/project (1x2=2)	22
		3x4=12		
	Or	Or	Or	
			Seminar (1x2=2)	
			Dissertation/Academic	
			Project/Entrepreneurship (1x20=20)	

# 6.1 Semester-wise Distribution of Discipline Specific Core (DSC) Courses:

D	Discipline Specific Core Courses (4 Credits each) Contact Hours					Credit
Dis	cipline Specific	<b>Elective Courses (4 Credits each)</b>	Con	tact H	ours	Credit
Semester	<b>Course Code</b>	Name Of The Course	L	T	P	
I	MAC401	Finite Field	3	1	0	4
I	MAC402	Topology	3	1	0	4
I	MAC403	Ordinary Differential Equation	3	1	0	4
II	MAC451	Functional Analysis	3	1	0	4
II	MAC452	Complex Analysis	3	1	0	4
II	MAC453	Operations Research	3	1	0	4
III	MAC501	Integral Equation and Calculus of Variations	3	1	0	4
III	MAC502	Applied Functional Analysis	3	1	0	4
IV	MAC551	Fuzzy Sets and Fuzzy Logics	3	1	0	4
IV	MAC552	Biomathematics	3	1	0	4

## **6.2 Details of Discipline Specific Elective (DSE) Courses:**

The Discipline Specific Electives (DSEs) are a pool of credit courses of Mathematics from which a student will choose to study based on his/ her interest. A student of Master of Science(M.Sc.) Mathematics gets an option of choosing one DSE of Mathematics in each of the semesters III and IV, while the student has an option of choosing a maximum of three DSE courses of Mathematics in these semesters. The semester wise distribution of DSE courses is listed in Table 4.

Disci	Discipline Specific Elective Courses (4 Credits each)		Cont	tact H	ours	Credit
Semester	<b>Course Code</b>	Name Of The Course	L	T	P	
Ι	MAE401	Mathematical Modelling	3	1	0	4
Ι	MAE402	Numerical Analysis	3	1	0	4
I	MAE403	Actuarial Mathematics	3	1	0	4
II	MAE451	Partial Differential Equation	3	1	0	4
II	MAE452	Linear Algebra	3	1	0	4
II	MAE453	Fractal Geometry and Dynamical System	3	1	0	4
III	MAE501	Fluid Dynamics	3	1	0	4
III	MAE502	Non-Linear Programming Problem	3	1	0	4
III	MAE503	Measure & Integration-1	3	1	0	4
III	MAE504	Applied Discrete Mathematics	3	1	0	4
IV	MAE551	Approximation theory	3	1	0	4
IV	MAE552	Measure & Integration-2	3	1	0	4
IV	MAE553	Project	0	0	4	4
IV	MAE554	Control Theory	3	1	0	4

## 6.3 Details of Skill Enhancement Courses (SECs)/Project:

D	Discipline Specific Core Courses (4 Credits each)  Contact Hours					
	Skill Enhance	ement Courses (SECs)/Project	Con	tact H	ours	Credit
Semester	<b>Course Code</b>	Name Of The Course	L	T	P	
I	MAS401	Seminar-1	0	0	2	2
II	MAS451	Seminar-2	0	0	2	2
III	MAS501	Seminar-3	0	0	2	2
IV	MAS551	Seminar-4	0	0	2	2

## **Course Details with Course-Specific Outcomes:**

# **Discipline Specific Core Course**

**MAC401: Finite Field** 

Course Prerequisites: An understanding of numbers, group, rings.

S.No.	Particular	Contact Hours
1.	Introduction of Groups, Rings, Fields, irreducible polynomials, roots of irreducible polynomials, primitive polynomials, construction of irreducible polynomials.	12
2.	Introduction to Galois Theory.	4
3.	Finite extensions, characterization of finite fields, Algebraic extensions, roots of polynomials, splitting fields.	8
4.	Separable extensions, Normal extensions, Algebraic closure, composite extensions, roots of unity.	8
5.	Cyclotomic Polynomial, cyclotomic extensions and abelian extensions over Q, representation of elements of finite fields.	10
	Total	42

### **Course Outcome:**

After studying this course the student will be able to:

- a) Identify and construct examples of fields, distinguish between algebraic and transcendental extensions, characterize normal extensions in terms of splitting fields and prove the existence of algebraic closure of a field.
- b) Characterize perfect fields using separable extensions, construct examples of automorphism group of a field and Galois extensions as well as prove Artin's theorem and the fundamental theorem of Galois theory.
- c) Classify finite fields using roots of unity and Galois theory and prove that every finite separable extension is simple.
- d) Use Galois theory of equations to prove that a polynomial equation over a field of characteristic is solvable by radicals iff its group (Galois) is a solvable group and hence deduce that a general quintic equation is not solvable by radicals.

- 1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley & Sons Inc., 3<sup>rd</sup> Ed., 2004.
- 2. W.W. Peterson and E.J. Weldon, Jr., Error-Correcting Codes. M.I.T. Press, Cambridge, Massachusetts, 1972.
- 3. S. Lang, Algebra, Springer (India) Pvt. Ltd., 2010.

- 4. R. Lidl and H. Niederreiter, Introduction to Finite Fields and their Applications, Cambridge University Press, 1994.
- 5. G.L. Mullen and C.Mummert, Finite Fields and Applications, Student Mathematical library, 41, AMS 2007. http://www.ams.org/bookstore-getitem/isbn=0-8218-4418-0
- 6. Khanna, V.K., Bhambri, S.K., A Course in Abstract Algebra, 4<sup>th</sup> Edition, Vikas Publishing House Pvt. Ltd., 2013.
- 7. Bhattacharya P.B., Jain S.K. and Hagpaul S.R., Basic Abstract Algebra Cambridge University Press, Second Edition

# MAC402: Topology

## **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Basics of Metric Spaces: Definitions and examples, Subspaces, Convergent sequences, Continuous and uniformly continuous functions, , Completeness, connectedness.	12
2.	Topological Spaces: Definition and basic concepts, Open bases and open subbases, weak topologies, compact spaces, Finite products, Locally compact spaces.	10
3.	Separability, Separation axioms, T <sub>1</sub> -spaces and Hausdorff spaces.	10
4.	Connectedness, component of space, totally disconnected spaces, Locally connected spaces.	10
	Total	42

#### **Course Outcome:**

After studying this course the student will be able to:

- a) The knowledge gained in this subject will make students able to generalize and extend concepts of real/complex analysis to more abstract spaces.
- b) It is expected that the critical reasoning ability will be further enhanced and sharpened at the end of the course. Students completing this course will be able to present mathematics clearly and precisely, make vague ideas precise.

This course will enable the students to understand the importance and properties of abstract analysis

## **Books Recommended:**

- 1. G.F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill Education Pvt. Ltd., New Delhi. 2004.
- 2. P.K. Jain and K. Ahmad, Metric Spaces, Second Edition, Narosa Publishing House, New Delhi, 2004.
- 3. J.R. Munkers, Topology, Prentice Hall, 1975.
- 4. W.J. Pervin, Foundations of General Topology, Academic Press, 1964.

# **MAC403: Ordinary Differential Equation**

S.No.	Particular	Contact	
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		Hours
1.	Introduction of differential Equations: Formation of differential equations.	12
	Basic definitions (linearity, homogeneous and non-homogeneous, explicit and	
	implicit solution general solution, &particular solution of First and second order	
	differential equations).	
	Existence and uniqueness theorem of Initial Value Problems: Picard's and	
	Peano's Theorems, Continuation of solution and maximal interval of existence, continuous dependence.	
2.	Linear systems, properties of homogeneous and non-homogeneous	6
	systems, behaviour of solutions of nth order linear homogeneous	
	equations.	
3.	Systems of ODEs. Phase Plane. Qualitative Methods: Two Dimensional	8
	Autonomous Systems: Phase Space Analysis, Constant-Coefficient Systems.	
	Phase Plane Method, Criteria for Critical Points. Stability, Eigen-value, proper	
	and improper nodes, spiral points and saddle points. Asymptotic behavior,	
	stability, Lyapunov methods. Qualitative Methods for Nonlinear Systems,	
	Nonhomogeneous Linear Systems of ODEs	
4.	Series Solution: Power series solution of second order homogeneous ODE,	8
	ordinary points, singular points, Frobenius series solution, Legendre and Bessel's	
	equation through examples	0
5.	Green's Function: Definition, Construction of Green's function for an important	8
	special case. Linear integral equations in cause and effect. The influence	
	function. Applications of Green's function. Eigen value problems. Self adjoint form, Sturm-Liouvile problem and itsapplications.	
	Total	42
	Total	76

After studying this course the student will be able to:

- a) know about existence, uniqueness and continuity of solutions of first order ODE's, properties of zeros of solutions of linear second order ODE's, boundary value problems.
- b) understandwitheigen values and eigen functions of Sturm-Liouville systems, and the solutions of initial and boundary value problems.

be well equipped to undertake any advanced course on ordinary differential equations.

### List of sample Practical by using software like MATLAB/MATHEMATICA/MAPLE/MAXIMA etc.

- 1. To solve first order linear differential equation.
- 2. To solve homogenous differential equations with constant coefficients.
- 3. To solve first order non-linear differential equations.
- **4.** To solve the first order initial value problems.
- **5.** To draw the phase lines for first order linear differential equations.
- **6.** To draw the phase lines for first order non-linear differential equations.
- 7. To find the equilibrium points of the autonomous.
- **8.** To find the critical points of the differential equation.
- **9.** To find the Eigen-values of the second order differential equations.
- 10. To find the saddle points of the differential equations.

- 1. Simmons, G. F., "Differential Equations", McGraw-Hill, 2nd Edition 1991.
- 2. Hildebrand F. B., "Methods of Applied Mathematics", Courier Dover Publications 1992
- 3. Tenenbaum, M. and Polard, H., "Ordinary Differential Equations", Dover Publications 1985
- 4. Sneddon, I. N., "Elements of Partial Differential Equations", McGraw-Hill Book Company 1988
- 5. Rao, K. S., "Introduction to Partial Differential Equations", PHI Learning Pvt. Ltd. (2nd Edition) 2010

**6.** Amarnath, T., "An Elementary Course in Partial Differential Equations", Narosa Publishing House (2nd Edition) 2012

# **MAC451: Functional Analysis**

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	Normed Spaces, Banach spaces, properties of normed spaces, Finite dimensional normed spaces and subspaces, Quotient normed spaces, Equivalent norms, Bounded and continuous linear operators, Linear functional, Finite dimensional normed spaces and compactness, Normed spaces of operators, Dual spaces.	12
2.	Inner product spaces, Hilbert spaces, properties of inner product spaces, orthogonal complements and Projection Theorem, Orthonormal sets and sequences, Total orthonormal sets and sequences.	10
3.	Representation Theorem for Hilbert spaces, Hilbert adjoint operator, self adjoint, unitary, normal and positive operators.	10
4.	Hahn Banach theorem for real/complex vector spaces and normed spaces, Adjoint Operators, reflexive spaces.	4
5.	Baire Category theorem, Uniform boundedness theorem, Open Mapping Theorem (OMT), Closed Graph Theorem.	6
	Total	42

#### **Course Outcome:**

After studying this course the student will be able to:

- a) Familiarize with distance and extension of distance as a norm. The student will learn central concepts from functional analysis, including the Hahn-Banach theorem, the open mapping and closed graph theorems and the uniform boundedness theorem.
- b) Structural properties of spaces those are constructed with the help of suitable norms. The students will be able to unify the various concepts of calculus and analysis with the help of other concepts from vector spaces, matrix theory and complex variables.
- c) The concepts of linear operators, their adjoint operators and properties of bounded linear operators. The significance of orthonormal sequences in the Fourier series and orthonormal expansions.

- 1. E.Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons (Asia) Pvt. Ltd., 2006.
- 2. J.B. Conway, A course in Functional Analysis, Second Edition Springer-Verlag, 2006.
- 3. P.K. Jain and O.P.Ahuja, Functional Analysis, Second Edition, New Age International Publication New Delhi, 2004.
- 4. G.F. Simmons, Introduction to Topology and Analysis, Tata McGraw Hill Education Pvt. Ltd., New Delhi, 2004.

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	Linen of half planes in complex plane, Extended complex plane, Stereographic projection.	6
2.	Power Series, Analytic functions, Analytic functions as mappings, Conformal mapping, Mobius transformations.	8
3.	Analytic Continuation: Direct Analytic Continuation, Monodromy theorem, Poisson Integral Formula, Analytical Formula, Analytical Continuation via Reflection.	06
4.	Taylor series, Power series representation of analytic functions, Zeroes of an analytic function, Index of a closed curve, Cauchy's theorem and integral formula, Homotopic version of Cauchy's theorem and simple connectedness, Counting zeroes, Open mapping theorem.	12
5.	Classification of singularities, Laurent series, Residues, Cauchy Residue theorem, Contour integration, Application to real integration, Argument principle, Maximum modulus theorem, Schwartz Lemma.	10
	Total	42

#### **Course Outcome:**

After studying this course the student will be able to

- a) understand analytic function as a mapping on the plane, Mobius transformation and branch of logarithm.
- b) understand Cauchy's theorems and integral formulas on open subsets of the plane.
- c) understand how to count the number of zeros of analytic function giving rise to open mapping theorem.
- d) know about the kind of singularities of meromorphic functions which helps in residue theory and contour integrations. handle integration of meromorphic function with zeros and poles leading to the argument principle and Rouche's theorem.

## **Books Recommended:**

- 1. J. Bak and D.J. Newman, Complex Analysis, Second Edition, Springer, 2011.
- 2. J.B. Conway, Function of One Complex Variable, Narosa, Delhi.
- 3. L.V. Ahlfors, Complex Analysis, McGraw Hill Co., New York, 1988.
- 4. J.W. Brown and R.V. Churchill, Complex Variables and Applications, McGraw Hill International Edition, 2009.
- 5. S. Lang, Complex Analysis, Springer-Verlag, 2003.

# **MAC453: Operations Research**

S.No.	Particular	Contact
		Hours

1.	Introduction to LPP( graphical, simplex, Big-M, 2-Phase, Dual-Simplex method	8
	etc.)	
2.	Assignment Problem and TP and Sequencing	6
3.	Game Theory: Two-person zero sum games, game with mixed strategies, graphical solution, and solution by linear programming.	7
4.	Replacement problems: Introduction, Replacement of Equipment, Replacement of Equipment that fails suddenly, Recruitment and promotion problem, Equipment renewal problem, Reliability and System Failure rates.	4
5.	Inventory, Features of Inventory system, Inventory Model Building, Deterministic Inventory Models with no shortage, Deterministic Inventory with shortages probabilistic Inventory Control Models: single period probabilistic model without setup cost-single period probabilities Model with setup cost. Queuing theory.	12
6.	Project Management by PERT/CPM, Network diagram, Rules of construction, Time estimate and critical path analysis, PERT.	5
	Total	42

After studying this course the student will be able to

- a) To appropriately formulate Linear Programming models for service and manufacturing systems, and apply operations research techniques and algorithms to solve these LP problems
- b) Apply linear programming method to solve two-person zero-sum game problems.
- c) To appropriately formulate Network models for service and manufacturing systems, and apply operations research techniques and algorithms to solve these Network problems.

To appropriately formulate Queuing and Inventory models for service and manufacturing systems, and apply operations research techniques and algorithms to solve these problems.

### **Books Recommended:**

- 1. Kanti-Swarup, P.K. Gupta and man-Mohan, Operations Research, S.Chand publication.
- 2. G.hadley: Linear Programming, Narosa publishing house 1995.
- 3. F.S. Hiller and G.J. Lieberman, Introduction to Operations Research (6<sup>th</sup> Ed.), Mc Graw Hill International Ed., 1995.
- 4. H.A Taha: Operations Research, An Introduction (3<sup>rd</sup> Ed.) Macmillan Co., New York, 1982.

### MAC501: Integral Equations & Calculus of Variation

S.No.	Particular	Contact Hours
1.	<b>Preliminary Concepts:</b> Definition and classification of linear integral equations. Conversion of initial and boundary value problems into integral equations. Conversion of integral equations into differential equations. Integrodifferential equations.	4
2.	<b>Fredholm Integral Equations:</b> Solution of integral equations with separable kernels, Eigenvalues and Eigenfunctions. Solution by the successive Approximate ons, Numann series and resolvent kernel. Solution of integral equations with symmetric kernels, Hilbert-Schmidt theorem, Green's function approach.	8
3.	Classical Fredholm Theory: Fredholm method of solution and Fredholm theorems	4

4.	Volterra Integral Equations: Successive approximations, Neumann series and	4
	resolvent kernel. Equations with convolution type kernels.	
5.	<b>Solution of integral equations by transform methods:</b> Singular integral equations, Hilbert-transform, Cauchy type integral equations.	6
6.	<b>Calculus of Variations:</b> Basic concepts of the calculus of variations such as functionals, extremum, variations, function spaces, the brachistochrone problem. Necessary condition for an extremum, Euler's equation with the cases of one variable and several variables, Variational derivative. Invariance of Euler's equations. Variational problem in parametric form.	10
7.	<b>General Variation:</b> Functionals dependent on one or two functions, Derivation of basic formula, Variational problems with moving boundaries, Broken extremals: Weierstrass – Erdmann conditions.	6
	Total	42

The student will be able to understand

- (a) the relationship between linear differential equations, Volterra and Fredholm integral equations, and their solutions by using resolvent kernels.
- (b) solution of Abel's integral equation, Iterated kernels, solution of Cauchy type integral equations, eigen values and eigen functions of Fredhold imntegral equations, Green's function.
- (c) Use of Laplace transformation to get the solution of integro-differential equations, Volterra integral equation of the First kind.

functionals, strong and weak variations, Euler's Equation, Variational problem, Isoperimetric problem.

#### **Books Recommended:**

- 1. Jerry, Abdul J., Introduction to Integral Equations with applications, Clarkson University Wiley Publishers (II Edition), 1999.
- 2. Chambers, Ll. G., Integral Equations: A short Course, International Text Book Company Ltd., 1976.
- 3. Kanwal R. P., Linear Integral Equations, BirkhäuserBosten, II Edition, 1997.
- 4. Harry Hochstadt, Integral Equations, John Wiley & Sons, 1989.
- 5. Gelfand, I. M., Fomin, S. V., Calculus of variations, Dover Books, 2000.
- 6. Weinstock Robert, Calculus of Variations with Applications to Physics and Engineering, Dover Publications, INC., 1974.

### **MAC502: Applied Functional Analysis**

## **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Applications of Banach Spaces and Hilbert Spaces. Summability of series.	12
	Banach fixed point theorem (BFPT). Existence of solutions of system of	
	linear equations using BFPT.	
2.	Weak and Weak* convergence. Numerical integration.	5
3.	Solutions of Differential and integral equations (Fredholm integral equation	9
	and Volterra integral equation).	
4.	Best Approximation in Banach and Hilbert spaces. Haar conditions and	14
	Criterion for uniqueness of best approximation.	
	Total	40

## **Course Outcome:**

(a) Learn the fundamental theorem, Hahn-Banach, Open mapping, Closed graph and Banach fixed point

theorems.

(b) Use the Banach fixed point theorem to demonstrate the existence and uniqueness of solutions to differential equations, Integral equations, numerical root finding, system of linear equations and eigen value problems.

#### **Books Recommended:**

- 1. S.Kesavan, Topics in Functional Analysis and applications, New Age International Publishers, Delhi, 1989.
- 2. E. Kreyszig, Introductory Functional Analysis with applications, John wiley& Sons (Asia) Pvt. Ltd., Singapore, 2010.
- 3. L.C. Evans, Partial differential equations, Graduate text in Mathematics, Vol. 19, AMS, 1998.
- 4. F.G. Friedlander, Introduction to the theory of distrutions, Cambridge University Press, Cambridge, 1982.
- 5. A.H. Zemanian, Distribution theory and transform analysis, McGraw Hill, New York, 1965.

## **MAC551: Fuzzy Sets and Fuzzy Logics**

## **Course Prerequisites:**

S.No.	Particular Particular	Contact
		Hours
1.	Fuzzy Sets and Uncertainty: Uncertainty and information, fuzzysets and	10
	membership functions, chance verses fuzziness, properties of fuzzy sets, fuzzy	
	set operations.Fuzzy Relations:Cardinality, operations, properties, fuzzy	
	Cartesian product and composition, fuzzy tolerance and equivalence	
	relations, forms of composition operation.	
2.	Fuzzy Logic and Fuzzy Systems: Classic and fuzzy logic,approximate	7
	reasoning, Natural language, linguistic hedges, fuzzyrule based systems,	
	graphical technique of inference.	
3.	Fuzzification and Defuzzification: Various forms of membershipfunctions,	10
	fuzzification, defuzzification to crisp sets and scalars. Development of	
	membership functions: Membership valueassignments: intuition, inference,	
	rank ordering	
4.	Fuzzy Arithmetic and Extension Principle: Functions of fuzzysets,	5
	extension principle, fuzzy mapping, interval analysis, vertexmethod and	
	DSW algorithm.	
5.	Fuzzy Optimization: One dimensional fuzzy optimization, fuzzyconcept	10
	variables and casual relations, fuzzy cognitive maps, agentbased	
	models.Fuzzy Control Systems:Fuzzy control system design problem,fuzzy	
	engineering process control, fuzzy statistical process control, industrial	
	applications.	
	Total	42

## **Course Outcome:**

The students will be able to understand

- (a) The basic ideas of fuzzy sets, arithmetic operations, norms and conforms, properties of fuzzy sets and fuzzy relations,
- (b) The basic features of membership functions, fuzzification process and different types of defuzzification methods,
- (c) Use of fuzzy logic, fuzzy inference systems and design of fuzzy rule based system along with fuzzy optimization.

#### **Books Recommended:**

- 1. Ross, T. J., "Fuzzy Logic with Engineering Applications", Wiley India Pvt. Ltd., 3rd Ed., 2011
- 2. Zimmerman, H. J., "Fuzzy Set theory and its application", Springer India Pvt. Ltd., 4th Ed., 2006.
- 3. Klir, G. and Yuan, B., "Fuzzy Set and Fuzzy Logic: Theory and Applications", Prentice Hall of India Pvt. Ltd., 2006.
- 4. Klir, G. and Folger, T., "Fuzzy Sets, Uncertainty and Information", Prentice Hall of India Pvt. Ltd., 2002
- 5. Kwang H. Lee, First course on Fuzzy Theory and applications, Springer.
- 6. T.J. Ross, Fuzzy Logic with Engineering Applications, Wiley.

#### **MAC552: Biomathematics**

## **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Mathematical Biology and the modeling process: an overview.	2
2.	Continuous models: Malthus model, logistic growth, Allee effect, Gompertz growth, Michaelis-Menten Kinetics, Holling type growth, Bacterial growth in a Chemostat, Harvesting a single natural population, Prey predator systems and Lotka Volterra equations, Populations in competitions, Epidemic Models (SI, SIR, SIRS, SIC), Activator-Inhibitor system, Insect Outbreak Model: Spruce Budworm, Numerical solution of the models and its graphical representation.	8
3.	Qualitative analysis of continuous models: Steady state solutions, stability and linearization, multiple species communities and Routh-Hurwitz Criteria, Phase plane methods and qualitative solutions, bifurcations and limit cycles with examples in the context of biological scenario.	10
4.	Spatial Models: One species model with diffusion, Two speciesmodel with diffusion, Conditions for diffusive instability, Spreadingcolonies of microorganisms, Blood flow in circulatory system, Traveling wave solutions, Spread of genes in a population.	8
5.	Discrete Models: Overview of difference equations, steady statesolution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Drug Delivery Problem, Discrete Prey-Predator models, Density dependent growthmodels with harvesting, Host-Parasitoid systems (Nicholson-Baileymodel), Numerical solution of the models and its graphicalrepresentation.	10
6.	Case Studies: Optimal Exploitation models, Models in Genetics, Stage Structure Models, Age Structure Models.	4
	Total	42

### **Course Outcome:**

- (a) Draw inferences from models using higher technique including problem solving, quantitative, qualitative using numerical methods.
- (b) Students gets the scientific &research ability by this multi-disciplinary course. Biological problem dealing by mathematics

#### **Books Recommended:**

1. Keshet, L. E., "Mathematical Models in Biology", SIAM, 1988.

- 2. Murray, J. D., "Mathematical Biology", Springer, 1993.
- 3. Fung, Y. C., "Biomechanics", Springer-Verlag, 1990.
- 4. Brauer, F., Driessche, P. V. D. and Wu, J., "Mathematical Epidemiology", Springer, 2008.
- **5.** Kot, M., "Elements of Mathematical Ecology", Cambridge University Press, 2001.

## MAC553: Project work

A project may be undertaken in the form of a case study or otherwise and data be collected, if required, as the case may be. The topic of the project be chosen in consultation with the assigned supervisor and the candidate should prepare a summary/synopsis of the proposed project related to some topic in Mathematics. The candidate needs to collect data/related literature on any particular aspect of the identified topic and shall prepare the report of the project from historical point of view, or as a survey or unification of different aspects.

# **Discipline Specific Elective (DSE) Courses**

## **MAE401: Mathematical Modelling**

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	What is Mathematical Modeling? History of Mathematical Modeling, latest development in Mathematical Modeling, Merits and Demerits of Mathematical Modeling.Introduction to difference equations, Non-linear Difference equations, Steady state solution and linear stability analysis. Introduction to Discrete Models, Linear Models, Growth models, Decay models, Newton's Law of Cooling, Bank Account Problem and mortgage problem.	06
2.	Compartment model, Drug Delivery Problem, Harrod Model of Economic growth, War Model, Lake pollution model, Alcohol in the bloodstream model, Arm Race models, Linear Prey-Predator models, Density dependent growth models with harvesting, Numerical solution of the models and its graphical representation using EXCEL.	12
3.	Introduction to Continuous Models, Carbon Dating, Drug Distribution in the Body, Growth and decay of current in a L-R Circuit, Horizontal Oscillations, Vertical Oscillations, Damped Force Oscillation, Dynamics of Rowing, Combat Models, Mathematical Model of Influenza Infection (within host), Epidemic Models (SI, SIR, SIRS, SIC), Spreading of rumor model, Steady State solutions, Linearization and Local Stability Analysis, logistic and gomperzian growth, prey-predator model, Competition models, Numerical solution of the models and its graphical representation using EXCEL.	12
4.	Mathematical Modelling through Ordinary Differential Equations of Second Order: Planetary Motions- Circular Motion and Motion of Satellites- Mathematical Modelling through Linear Differential Equations of Second Order- Miscellaneous Mathematical Models. Mathematical.	8
5.	Modelling through Difference Equations: Simple Models- Basic Theory of linear difference equations with constant coefficients- Economics and Finance-Population Dynamics and Genetics-Probability Theory.	4
	Total	42

### **Course Outcome:**

After studying this course the student will be able to:

a) Know the concept of how to develop mathematical models using experimental data and observation data (Discrete and continuous) in the form of difference and differential equations and basic methods to solve them.

To develop Models of growth and decay (for problems of aging), Prey Predator (problems of ecology and environment), drugs delivery problem (Medical problems), motion of planets and satellites (Space problems), population dynamics (genetics and microbiology), etc.

## **Books Recommended:**

- 1. Albright, B., "Mathematical Modeling with Excel", Jones and Bartlett Publishers 2010
- 2. Marotto, F. R., "Introduction to Mathematical Modeling using Discrete
- 3. Dynamical Systems", Thomson Brooks/Cole. 2006
- 4. Kapur, J. N., "Mathematical Modeling", New Age International 2005
- **5.** Barnes, B. and Fulford, G. R., "Mathematical Modelling with Case Studies", CRC Press, Taylor and Francis Group. 2009

Edsberg, L., "Introduction to Computation and Modeling for Differential Equations", John Wiley and Sons. 2008

## **MAE402: Numerical Analysis**

## **Course Prerequisites:** Basic mathematics.

S.No.	Particular	Contact Hours
1.	Revisit of root finding methods for nonlinear equations and their order of convergence.	6
2.	<b>System of Non-linear Equations:</b> Newton's method, Quasi-Newton methods, Broyden's method, Applications-coupled reversible chemical reaction, flow distribution in a pipe flow network.	10
3.	<b>Interpolation:</b> Piecewise interpolation, cubic spline interpolation, Hermite interpolation, Hermite cubic interpolation.	6
4.	<b>Initial Value Problems:</b> Euler's method, Higher order Tylor's method, Runge-Kutta methods, multistep methods such as Adam-Bashforth and Adam-Moulton methods, Convergence and Stability, Applications-spread of an epidemic, radiative heat transfer to a thin metal plate, geneting switch.	10
5.	Two-Point Boundary Value Problems: Finite difference methods for linear problems with Dirichlet as well as non-Dirichlet boundary conditions, Applications-flow between parallel plates, the heat pack.	10
	Total	42

#### **Course Outcome:**

The course will enable the students to

- a) Learn some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
- b) Interpolation techniques to compute the values for a tabulated function at points not in the table, spline. Numerical solution of differential equations using single step and multi step methods along with their stability analysis.

## $List\ of\ sample\ Practical\ by\ using\ software\ like\ MATLAB/MATHEMATICA/MAPLE/MAXIMA\ etc.$

- (i) Solve a system of 2 (and 3) nonlinear equations of 2 (and 3) variable using Newton's method.
- (ii) Solve a system of 2 (and 3) nonlinear equations of 2 (and 3) variables using Broyden's method.
- (iii) Solve the system of nonlinear equations arising from the problem of coupled reversible chemical reactions using Newton's method as well as using Broyden's method.
- (iv) Solve the system of nonlinear equations arising from the problem of flow distribution in a pipe flow network using Newton's method as well as using Broyden's method.
- (v) For a given data, calculate the value at a given point of the generated Hermite interpolating polynomial.

- (vi) Tabulate the approximate solutions of an initial value problem (IVP) for different given step sizes using Euler's method and second order Runga-Kutta method.
- (vii) Repeat the above for the IVP arising from the problems of the spread of an epidemic, radiative heat transfer to a thin metal plate and geneting switch.
- (viii) Tabulate the approximate solutions of a two-point boundary value problem (BVP) for different given step sizes.
- (ix) Repeat the above for the BVP arising from the problems of the flow between parallel plates.
- (x) Repeat the above for the BVP arising from the problems of the heat pack.

#### **Books Recommended:**

- 1. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, Inc., 2006.
- 2. K. Atkinson and W.Han, Elementary Numerical Analysis, third Edition, Willey India (P) Ltd., 2004.
- 3. C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Seventh Edition, Pearson Education, Inc., 2004.
- 4. Froberg C.E., Introduction to Numerical Analysis Addition Wesley, Second Edition, 1969.

#### **MAE403: Actuarial Mathematics**

## **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Basic Principles: comparison, arbitrage and risk aversion, Interset (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond price and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, putable and callable bonds.	10
2.	Assent return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowits model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharp index. Capital Asset Pricing Model (CAPM), beats of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.	12
3.	Forwards and futures, marking to market, value of a forward/futures contract, replacing portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps.	10
4.	Lognormal distribution, Lognormal model/Geometric Brownian Motion for stock prices, Binomial Tree model for stock prices, parameter estimation, comparison of the models. Options, Types of Options: put/call, European/American, pay off of an option, factors affecting option prices, put call parity.	10
	Total	42

#### **Course Outcome:**

### **Books Recommended:**

- 1. David G. Luenberger, Investment Science, Oxford University Press, Delhi, 1998.
- 2. John C. Hull, Options, Futures and Other Derivatives (6<sup>th</sup> Ed.) Prentice-Hall India, Indian reprint, 2006.
- 3. Sheldon Ross, An Elementary Introduction to Mathematical Finance (2<sup>nd</sup> Ed.), Cambridge University Press, USA, 2003.

## **MAE451: Partial Differential Equations**

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	<b>Introduction:</b> Surfaces and curves. Simultaneous differential equations of the first order and first degree. Integral curves of vector fields. Methods of solution of $dx/P = dy/Q = dz/R$ . Orthogonal Trajectories of a system of curves on a surface. Pfaffian differential forms and equations. Solution of Pfaffian differential equations in three variables.	6
2.	<b>First Order PDE:</b> Partial differential equations, Origins and classification of first order PDE, Initial value problem for quasi-linear first order equations: Existence and uniqueness of solution, Nonexistence and non-uniqueness of solutions. Surfaces orthogonal to a given system of surfaces. Nonlinear PDE of first order, Cauchy method of characteristics, Compatible systems of first order equations, Charpit's method, Solutions satisfying given conditions. Jacobi's method.	8
3.	<b>Second Order PDE:</b> The origin of second order PDE. Equations with variable coefficients, Classification and canonical forms of second order equations in two variables. Classification of second order equations in n variables. Characteristic curves of second order equations in two variables. Importance of characteristic curves.	5
4.	Review of Integral Transform and Fourier series.	2
5.	<b>Hyperbolic Equations:</b> Derivation of one and two dimensional, Wave, Heat equation, diffusion equation in reactions, initial & boundary value problem and their fundamental solution, wave equation: uniqueness, D'Alembert's solution of wave equations	9
7.	Elliptic Equations: Laplace equation in Cartesian, polar, spherical and cylindrical coordinates and its solution by Fourier series method,  Poisson equation in 2D. weak and strong maximum principle, Dirichlet's principle, existence of solution using Perron's method.  Parabolic Equations: solution of homogeneous and non-homogeneous	5
	diffusion equation (1D). Duhamel's principle.	
	Total	42

## **Course Outcome:**

After studying this course the student will be able to

- a) Establish a fundamental familiarity with partial differential equations.
- b) Distinguish between linear and nonlinear partial differential equations.
- c) Find complete integrals of Non-linear first order partial differential equations.

Solve second order partial differential equations by method of characteristic equations, and by method of separation of variables (Elliptic, parabolic, and Hyperbolic).

## List of sample Practical by using software like MATLAB/MATHEMATICA/MAPLE/MAXIMA etc.

- (i) To solve one-dimensional heat equation with initial values.
- (ii) To solve one -dimensional wave equation with initial values.
- (iii) To solve Laplace equation with initial values.
- (iv) To solve two-dimensional heat equation with initial values.
- (v) To solve two-dimensional wave equation with initial values.
- (vi) To draw 3D plot of the solution of heat equation.
- (vii) To draw 3D plot of the solution of wave equation.
- (viii) To draw 3D plot of the solution of Laplace equation.
- (ix) To draw the contour plot of solution of Laplace's equation in 2D.
- (x) To classify the seconder order partial differential equations.

The scope of the course is indicated by the relevant sections of chapter 10, 11, 13 of [1].

#### **Books Recommended:**

- 1. Simmons, G. F., "Differential Equations", McGraw-Hill, 2nd Edition 1991.
- 2. Hildebrand F. B., "Methods of Applied Mathematics", Courier Dover Publications 1992.
- 3. Tenenbaum, M. and Polard, H., "Ordinary Differential Equations", Dover Publications 1985.
- 4. Sneddon, I. N., "Elements of Partial Differential Equations", McGraw-Hill Book Company 1988.
- 5. Rao, K. S., "Introduction to Partial Differential Equations", PHI Learning Pvt. Ltd. (2nd Edition) 2010.
- 6. Amarnath, T., "An Elementary Course in Partial Differential Equations", Narosa Publishing House (2nd Edition) 2012.

#### MAE452: Linear Algebra

### **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Vector Spaces: Vector space, subspace, sum of subspaces, linear	12
	combination, linear dependence and independence, basis and dimension, examples of infinite dimensional spaces, ordered bases and coordinates.	
2.	Linear Transformation: Basic definitions, rank-nullity theorem, matrix	8
	representation, algebra of linear transformations, change of basis, linear	
	functional, Dual Spaces.	
3.	Canonical Forms: Eigen-values of linear operators, Eigen-space, minimal polynomial, diagonalization, invariant subspaces, Jordan canonical representation, Norm of a matrix, computation of a matrix Exponential.	10
4.	Inner Product Space: Definition of inner product between two vectors, orthogonal and orthonormal vectors, normed space, Gram-Schmidt process for orthogonalization, projection operator, quadratic forms, positive definite forms, Symmetric, Hermitian, orthogonal, unitary and Normal transformations/matrices.	12
	Total	42

#### **Course Outcome:**

After studying this course, the student will be able to:

- a) Learn about the concept of linear independence of vectors over a field, and the dimension of a vector space.
- b) Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation, and the change of coordinate matrix.
- c) Compute the characteristic polynomial, eigenvalues, eigenvectors, and eigenspaces, as well as the geometric and the algebraic multiplicities of an eigenvalue and apply the basic diagonalization result.

Compute inner products and determine orthogonality on vector spaces, including Gram-Schmidt orthogonalization to obtain orthonormal basis.

#### **Books Recommended:**

- 1. Hoffman, K. and Kunze, R., "Linear Algebra", 2nd edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India 2004
- 2. Leon, S.J., "Linear Algebra with Applications", 8th Edition, Pearson 2009
- 3. Peter, J. Olever and Shakiban, C., "Applied Linear Algebra", 1st Edition, Prentice Hall 2005
- 4. Strang, G., "Linear Algebra and its Applications", 3rd edition, Thomson Learning Asia Pvt Ltd 2003
- 5. Sudan L., "Applied Linear Algebra", Prentice Hall 2001

## MAE453: Fractal Geometry and Dynamical System

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	Measure and Mass distributions, Hausdorff measure, Hausdorff dimension,	8
1.	calculation of dimension (simple cases), box-counting dimension and its	· ·
	properties, calculation of box counting dimension in simple cases.	
2.	Techniques for calculating dimensions: basic methods, mass distribution	8
	principle, some examples, uniform cantor sets, covering lemma. Iterated	
	function schemes, dimension of self-similar sets, calculation for Sierpinski	
	gaskets, modified Von-Knock curve.	
3.	Some variations, non-linear Cantor set, self-affine sets, applications to	7
	encoding images, dynamical systems, repellers and iterated function schemes,	
	logistic map.	
4.	Theory of Julia sets: iteration of complex functions, Julia sets and its properties,	11
	quadratic functions, Mandelbrot set, Julia sets of quadratic functions.	
	Total	42

#### **Course Outcome:**

#### **Books Recommended:**

- 1. F.Kenneth, Factal Geometry, Mathematical Foundations and Applications, John Willey & Sons, 1995.
- 2. M.F. Barnsley, Fractal Everywhere, Acedemic Press, 1988.
- 3. J. Feder, Fractals, Plenum Press, New York, 1988.
- 4. B.B. Mandelbort, The Fractal Geometry of Nature, Freeman, San Francisco, 1982.
- 5. H.O.Peitgen and P.H. Richter, The Beauty of Fractals, Springer, Berlin, 1986.

**MAE501: Fluid Dynamics** 

S.No.	Particular	Contact Hours
1.	Lagrangian and Eulerian descriptions, Continuity of mass flow, circulation,	6
	rotational and irrotational flows, boundary surface, streamlines, path lines, streak lines, vorticity	
2.	General equations of motion: inviscid case, Bernoulli's theorem, compressible and incompressible flows, Kelvin's theorem, constancy of circulation.	4
3.	Stream function, Complex-potential, source, sink and doublets, circle theorem, method of images, Theorem of Blasius, Strokes stream function, Motion of a sphere.	5
4.	Helmholtz's vorticity equation, vortex filaments, vortex pair.	2
5.	Dimensional analysis, large Reynold's numbers; Laminar boundary layer equations, Similar solutions; Flow past a flat plate, Momentum integral equations, Solution by Karman-Pohlhausen methods, impulsive flow Reyleigh problem, dynamical similarity Thermal boundary layer equation for incompressible flow; Temperature distribution in Coutte flow and in flow past a flat plate.	9
6.	Navier-Stokes equations, dissipation of energy, diffusion of vorticity, Steady flow between two infinite parallel plates through a circular pipe (Hagen-Poiseuille flow), Flow between two coaxial cylinders, Energy equation, Dynamical similarity.	5
7.	Mathematical formulation of the stability problem of incompressible flow, Stability of flows under different cases, Prandtl's momentum transfer theory.	7
8.	Introduction to complex fluid.	4
	Total	42

## **Books Recommended:**

- 1. Batechelor, G.K., "An Introduction to Fluid Dynamics", Cambridge Press, 2002.
- 2. Schliting, H., Gersten K., "Boundary Layer Theory", Springer, 8th edition, 2004.
- 3. Rosenhead, "Laminar Boundary Layers", Dover Publications, 1963.
- 4. Drazin, P.G., Reid W. H., "Hydrodynamic Stability", Cambridge Press, 2004.
- 5. W.H. Besant and A.S. Ramsay, A Treatise on Hydromechanics, Bell publisher, 1913.

## **MAE502: Non-Linear Programming Problem**

S.No.	Particular	Contact Hours
1.	Convex Functions, Karash Kuhn-Tucker Theory, Convex	12
	QuadraticProgramming, Wolfe's, Beale and Pivot Complementary	
	Algorithm,Separable Programming.	
2.	Geometric Programming: Problems with positive co-efficient up-toone degree	6
	of difficulty, Generalized method for problems withpositive and negative	
	coefficients.	
4.	Dynamic Programming: Discrete and continuous DynamicProgramming,	6
	Simple illustrations.	
5.	Search Techniques: Direct Search and Gradient Methods, UnimodalFunctions,	11
	Fibonacci Method, Golden Section Method, Method ofSteepest Descent,	

	Newton Raphson Method, Hookes and JeevesMethod, Conjugate Gradient Methods.	
6.	Constrained optimization: Penalty function approach, Barrier Function Approach.	2
7.	Multi-objective and Goal Programming.	5
	Total	42

- (a) Understand the basic concepts of NLPP, difference between LPP and NLPP, convex set and convex function, relative maxima and relative minima.
- (b) Solution of NLPP problems with equality constraints using Lagrange multiplier method, solution of NLPP problem with inequality constraint using Karush Kuhn-Tucker(KKT) conditions.
- (c) Different types of numerical optimization methods, dynamic programming, geometric programming, and separable programming.

### **Books Recommended:**

- 1. Kanti-Swarup, P.K. Gupta and Man-Mohan, Operations Research, S.Chand publication.
- 2. G.hadley: Linear Programming, Narosa publishing house 1995.
- 3. F.S. Hiller and G.J. Lieberman, Introduction to Operations Research (6<sup>th</sup> Ed.), Mc Graw Hill International Ed., 1995.
- 4. H.A Taha: Operations Research, An Introduction (3<sup>rd</sup> Ed.) Macmillan Co., New York, 1982.
- 5. Mohan C., Deep, K., "Optimization Techniques", New Age India Pvt. Ltd, New Delhi, 2009.
- 6. Bazaraa, M., Sherali, H. D. and Shetty, C. M., "Nonlinear Programming: Theory and Algorithms", Wiley-Interscience; 3rd Ed., 2006.
- 7. Himmelblau, D. M., "Applied Nonlinear Prograaming", Mcgraw-Hill, 1972.
- 8. Mittal K.V. and Mohan C., Optimization Methods in System Analysis & O.R. New Age India Pvt. Ltd, New Delhi, 2009.

#### MAE503: Measure and Integration-1

S.No.	Particular	Contact Hours
1.	Countable sets, uncountable sets, Cardinal numbers of the sets of natural numbers, the set of real numbers and the set of functions, Schroeder-Bernstein theorem, Cantor's theorem, Order relation between cardinal numbers, Algebra and Exponentiation of cardinal numbers, the extended real numbers, Borel sets.	8
2.	Ideal measure, Outer measure and its properties, Measure spaces, Algebra of measurable sets. Outer measure, Lebesgue measure, Approximation of outer measure of a set, Measurability of Borel sets, Non-measurable sets.	10
3.	Measurable functions, Equivalence of various criteria for measurability, Littlewood's three principles, Approximation of Measurable Functions by Continuous Functions, Lusin's Theorem, Vitali-Caratheodory Theorem, Egoroff's Theorem.	10
4.	Lebesgue Integral: Simple functions, integral of a simple function, Lebesgue integral of a bounded measurable function over a set of finite measure,	12

Comparison of Riemann and Lebesgue integrals, Theorem of bounded convergence, The integral of a non-negative measurable function, Fatou's Lemma, Monotone convergence theorem, The general Lebesgue integral, Lebesgue's theorem of dominated convergence. Convergence in measure, Convergence almost everywhere.	
Total	40

After studying this course, the student will be able to

- a) Verify whether a given subset of P or a real valued function is measurable
- b) Understand the requirement and the concept of the Lebesgue integral (a generalization of the Reimann integration) along its properties.
- c) Demonstrate understanding of the statement and proofs of the fundamental integral convergence theorems and their applications.
- d) Know about the concepts of functions of bounded variations and the absolute continuity of functions with their relations.
- e) Extend the concept of outer measure in an abstract space and integration with respect to a measure.
- f) Learn and apply Holder and Minkowski inequalities in Lp-Spaces and understand completeness of Lp-Spaces and convergence in measures.

#### **Books Recommended:**

- 1. H.L.Royden, Real Analysis, (3<sup>rd</sup> ed.), The Macmillian Company, New York, 1988.
- 2. G.de Berra, Introduction to Measure Theory, Van Nostrand Reinhold Company, New York, 1974.
- 3. R.G. Bartle, The Elements of Integration and Lebesgue Measure, John Wiley & Sons, Inc. New York, 1995.
- 4. P.K.Jain, V.P. Gupta and P. Jain, Lebesgue Measure and Integration, (2<sup>nd</sup> ed.), New Age International Publishers, New Delhi, 2011.
- 5. J.N. McDonald and N.A. Weiss, A Course in Real Analysis, Academic Press, New York, 1999.

## **MAE504: Applied Discrete Mathematics**

S.No.	Particular	Contact
		Hours
1.	Definition, example and basic properties of graphs, pseudo-graphs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.	10
2.	The coding problem, types of codes, Block codes, Error-detecting and error-correcting codes, Linear codes, The Hamming Metric, Description of linear block codes by matrices, dual codes, standard array, syndrome, Plotkin bound.	10
3.	Hamming sphere packing bound, Varshamov-Gilbert-Sacks bound, Hamming codes.	6
4.	Cyclic codes, Cyclic codes as ideals, Matrix description of cyclic codes, Hamming codes as cyclic codes, Bose-Chaudhary-Hocquenghem (BCH) codes, Reed-Solomon (RS) codes.	6
5.	RSA, attacks on RSA, Diffie-Hellman key exchange, EIGamal public key cryptosystem, cryptographic hash function, RSA signature, EIGamal signature.	10
	Total	42

#### **Books Recommended:**

- 1. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory (2<sup>nd</sup> Edition), Pearson Education (Singapore) Pvt. Ltd., Indian Reprint, 2003.
- 2. Raymond Hill, A First Course in Coding theory, Oxford University Press, 1990.
- 3. W.W. Peterson and E.J. Weldon, Jr. Error-Correcting Codes. M.I.T. Press, Cambridge, Massachusetts, 1972.
- 4. Johannes A. buchmann, Introduction to Cryptography, Sprinnger 2000.

## **MAE551: Approximation Theory**

# **Course Prerequisites:**

S.No.	Particular	Contact
		Hours
1.	Concept of best approximation in a normed linear space, Existence of the best approximation, Uniqueness problem, Convexity-uniform convexity, strict convexity and their relations, Continuity of the best approximation operator.	10
2.	The Weierstrass theorem, Bernstein polynomials, Korovkin theorem, Algebraic and trigonometric polynomials of the best approximation, Lipschitz class, Modulus of continuity/ Integral modulus of continuity and their properties, Rate of approximation.	10
3.	Bernstein's inequality, Jackson's theorems and their converse theorems, Approximation by means of Fourier series.	12
4.	Positive linear operators, Monotone operators, Simultaneous approximation, $L^p$ -approximation, Approximation of analytic functions.	10
	Total	42

### **Course Outcome:**

- (a) The students will be able to recognize problems of engineering, science in abstract setting and construct the suitable mathematical model.
- (b) The confluence of abstract, applied mathematics and computer softwares will lead to efficiently understand and solve the problems. The convergence of approximation methods and possibility of approximation will be learn.
- (c) Learn the existence and uniqueness problems and dependence of the solution on structure of the space as well as the type of approximation.

- 1. E. W. Cheney, E. W., "Introduction to Approximation Theory", AMS Chelsea Publishing Co., 1981.
- 2. Lorentz, G. G., "Bernstein Polynomials", Chelsea Publishing Co., 1986

- 3. Natanson, I. P., "Constructive Function Theory Volume-I", Fredrick Ungar Publishing Co., 1964.
- 4. Mhaskar, H. M. and Pai, D. V., "Fundamentals of Approximation Theory", Narosa Publishing House, 2000.
- 5. Timan, A. F., "Theory of Approximation of Functions of a Real Variable", Dover Publication Inc., 1994.

## **MAE552: Measure and Integration-2**

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	Differentiation and Integration: Dini derivatives, Differentiation of monotone functions, Vitali's covering theorem, Lebesgue Theorem, Differentiation of an integral,	8
2.	Functions of bounded variation, Results on total variation, Jordan Decomposition Theorem, Absolutely continuous functions, Existence of an anti-derivative, theorem that a function is an indefinite integral if and only if it is absolutely continuous. Properties of indefinite integrals. Indefinite integrals of absolutely continuous functions.	12
3.	The L <sup>p</sup> -Spaces, Holder and Minkowski inequalities, Riesz-Fischer theorem on Completeness of L <sup>p</sup> -Spaces, Separability of L <sup>p</sup> -Spaces, Approximation of <i>p</i> -absolutely integrable function, density results, convergence in the mean, Bounded linear functionals in L <sup>p</sup> -Spaces.	12
4.	Product measure, Products of $\sigma$ -algebras and measures, Lebesgue measure on $R^2$ and its properties, Monotone Class Theorem, Products of Measures, Tonelli's Theorem, Fubini's Theorem	8
	Total	40

#### **Course Outcome:**

After studying this course the student will be able to

- a) Verify whether a given subset of P or a real valued function is measurable
- b) Understand the requirement and the concept of the Lebesgue integral (a generalization of the Reimann integration) along its properties.
- c) Demonstrate understanding of the statement and proofs of the fundamental integral convergence theorems and their applications.
- d) Know about the concepts of functions of bounded variations and the absolute continuity of functions with their relations.
- e) Extend the concept of outer measure in an abstract space and integration with respect to a measure.
- f) Learn and apply Holder and Minkowski inequalities in Lp-Spaces and understand completeness of Lp-Spaces and convergence in measures.

- 1. H.L.Royden, Real Analysis, (3<sup>rd</sup> ed.), The Macmillian Company, New York, 1988.
- 2. G.de Berra, Introduction to Measure Theory, Van Nostrand Reinhold Company, New York, 1974.
- 3. R.G. Bartle, The Elements of Integration and Lebesgue Measure, John Wiley & Sons, Inc. New York, 1995.
- 4. P.K.Jain, V.P. Gupta and P. Jain, Lebesgue Measure and Integration, (2<sup>nd</sup> ed.), New Age International Publishers, New Delhi, 2011.
- 5. J.N. McDonald and N.A. Weiss, A Course in Real Analysis, Academic Press, New York, 1999.

#### **MAE553: Project (based on some scientific problem)**

Any topic in mathematics may be picked up by a candidate in consultation with the assigned supervisor. An in-depth study of the topic in a specific direction be made leading to the identification of a problem. The derivation of full/partial answer to the problem be written in the form of a thesis. The investigation be made either to give birth to another proof of an existing result or a new technique be proposed in lieu of an existing technique or a novel finding.

## **MAE554: Control Theory**

## **Course Prerequisites:**

S.No.	Particular	Contact Hours
1.	Mathematical models of control systems, State space representation, Autonomous and non- autonomous systems, State, transition matrix, Peano series Solution of linear dynamical system.	4
2.	Block diagram, Transfer function, Realization, Controllability, Kalman theorem, Controllability Grammian, Control computation using Grammian matrix, Observability, Duality theorems., Discrete control systems, Controllability and Observability results for discrete systems.	10
3.	Companion form, Feedback control, State observer, Realization	6
4.	Liapunov stability, Stability analysis for linear systems, Liapunov theorems for stability and instability for nonlinear systems, Stability analysis through Linearization, Routh criterion, Nyquist criterion, Stabilizability and detachability,	8
5.	State feedback of multivariable system, Riccatti equation, Calculus of variation, Euler- Hamiltonian equations, Optimal control for nonlinear control systems, Computation of optimal control for linear systems.	8
6.	Control systems on Hilbert spaces, Semi group theory, Mild solution, Control of a linear system	6
	Total	42

## **Course Outcome:**

- 1. Burghes, D. N. and Graham, A., Introduction to control Theory including optimal control, John Wiley & Sons., 1980.
- 2. Canon, M. D., Culum, J.R., CC and Polak E., Theory of optimal control and Mathematical Programming, McGraw Hill, 1970.
- 3. Kirk, D.E., Optimal control theory-An introduction, Prentice Hall, 1970.
- 4. Lee, E. G., Markus L., Foundations of Optimal control theory, John Wiley & Sons, 1967.
- 5. Hull, D.G., Optimal control theory, Springer, 2005.
- 6. Geering, H. P., "Optimal Control with Engineering Applications", Springer, 2007.

# **SEC/Project/ Dissertation**

MAS401: Seminar-1 MAS451: Seminar-2 MAS501: Seminar-3 MAS551: Seminar-4

The topic of the seminar be chosen in consultation with the assigned supervisor and the candidate should prepare a summary/synopsis/presentation of the proposed seminar related to some topic in Mathematics.