Doon University, Dehradun

Sample Paper

M.Sc. Mathematics

Roll Number					
Programme Name					
Examination Centre					
Date of Examination					

Signatures of Candidate	Name of the Invigilator	Signature of the Invigilator

Time Allowed: 2 Hours ***Maximum Marks: 100***

INSTRUCTIONS FOR CANDIDATES

Candidates must read carefully the following instructions before attempting the Question Paper.

(i) Write your Roll Number in the space provided above

(ii) There are 50 questions. Attempt all.

(iii) Use ONLY BLUE/BLACK Ballpoint Pen to tick the correct option. Do not use Pencil.

(iv) Please do not make any stray marks on the Answer Sheet.

(v) Please do not do any rough work on the Answer Sheet.

(vi) Each question carries 2 mark. There will be no negative marking.

(vii) Pages at the end have been provided for rough work.

(viii) All answers must be tick marked directly on the question paper. Mark your answer **only inside the box** given against the options as follows.

a.	
b	√
c.	
d.	

1. If a square matrix of order 10 has exactly 5 distinct eigen values, then the degree of the minimal polynomial is

a.	At least 5	
b.	At most 5	
c.	Always 5	
d	Exactly 10	

2. What is the identity element for a group (z,+) - the set of all integers under the operation addition

a.	0	
b.	1	
c.	-1	
d	all of the above	

3. Let (H_1, .) & (H_2,.) are the two subgroups of the group (G, .). Which of the statement below is true.

a.	($H_1 \cap H_2$, .) is a group	
b.	($H_1 \cup H_2$, .) is a group	
c.	both are true	
d	none of the above	

4. The following pseudocode finds a real root of $f(x) = x^3 + x^2 - 36$ with error at most 0.0001.

Input: x
Output: x, y, y'
REPEAT
$y := x^3 + x^2 - 36$
$y' := 3x^2 + 2x$
$x := x - y/y'$
UNTIL y < .0001

which of the following methods is being used in this code

a.	Newton's iteration method : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	
b.	Viète trigonometic method	
c.	Method of Hermite	
d	None of the above	

5. Let **A** be a $n \times n$ matrix. Then:

a.	Rank(**A**) < n	
b.	Rank(**A**)≤ n	
c.	Rank(**A**) = n	
d	Rank(**A**) > n	

6. Let $\boldsymbol{V}$ be an n-dimensional space over a field **F**

a.	A subset having n non-zero vectors forms the basis for $\boldsymbol{V}$	
b.	A subset having n vectors which generates $\boldsymbol{V}$ forms the basis for $\boldsymbol{V}$	
c.	A subset having n distinct vectors forms the basis for $\boldsymbol{V}$	
d	A subset which is linearly independent forms the basis for $\boldsymbol{V}$	

7. Every odd degree polynomial with:

a.	Rational coefficients has a rational root	
b.	Integer coefficients has a rational root	
c.	Rational coefficients has an integer root	
d	Integer coefficients has a real root.	

8. A random variable X is uniformly distributed between 2 and 5. Its probability density function is:

$$f_X(x) = \begin{matrix} c & if\ x \in [2,5] \\ 0 & if\ x \notin [2,5] \end{matrix}$$

where c is a constant. $f_X(x)$ is a legitimate probability density function if:

a.	c = 3	
b.	c = 1/5	
c.	c = 1	
d	c = 1/3	

9. Let X be an absolutely continuous random variable having a standard normal distribution. Let:

$$Y = 3 + 2X$$

The probability density function of Y is

a.	$f_Y(y) = (2\pi)^{-2} exp(-\frac{1}{2}(y-3)^2)$	
b.	$f_Y(y) = (2\pi)^{-2} exp(-2(y-3)^2)$	
c.	$f_Y(y) = (8\pi)^{-2} exp(-\frac{1}{8}(y-3)^2)$	
d	$f_Y(y) = (\pi)^{1/2} exp(-\frac{1}{4}(y+3)^2)$	

10. Time-series analysis is based on the assumption that

a.	Random error terms are normally distributed	
b.	There are dependable correlations between the variable to be forecasted and other independent variables.	
c.	Past patterns in the variable to be forecasted will continue unchanged into the future	
d	The data do not exhibit a trend.	

11. The purchase cost is 30,000 and the depreciation is 5,000 then the depreciation function is

a.	V = f(t) = 30000 - 5000t	
b.	V = f(t) = 5000t + 30000	
c.	V = f(t) = 30000t - 5000t	
d	V = f(t) = 30000t + 5000t	

12. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$, is

a.	$3y = 9x + 2$	
b.	$y = 2x + 1$	
c.	$2y = x + 8$	
d	$y = x + 2$	

13. If the focus of a parabola is (–2, 1) and the directrix has the equation $x + y = 3$, then the vertex is

a.	(0,3)	
b.	(-1,1/2)	
c.	(-1,2)	
d	(2,-1)	

14. Equation of asymptotes of the hyperbola $xy - 3x - 4y + 8 = 0$, is

a.	$x = 3, y = 4$	
b.	$x = 0, y = 0$	
c.	$x = 4, y = 3$	
d	none of these	

15. The general solution of a differential equation is $(y + c)^2 = cx$, where c is an arbitrary constant. The order and degree of the differential equation are respectively

a.	*1,2*	
b.	*2,2*	
c.	*1,1*	
d	2,1	

16. Area of region satisfying $x \le 2$, $y \le |x|$ and $y \ge 0$ is

a.	1 sq. units	
b.	4 sq. units	
c.	2 sq. units	
d	none of these	

17. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n+1} =$

a.	e	
b.	$1/e$	
c.	$2e$	
d	$2/e$	

18. Let $\{s_n\}$ be a monotonic sequence. Then $\{s_n\}$ is convergent if and only if it is

a.	Bounded	
b.	Unbounded	
c.	Alternating	
d	None of the above	

19. The series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \ldots$ is

a.	Convergent	
b.	Divergent	
c.	Can't be determined	
d	None of the above	

20. If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$

a.	r	
b.	$\sin\theta$	
c.	$\cos\theta$	
d	$r.\tan\theta$	

21. $\int_0^{\frac{\pi}{2}} \sin^4 x.\cos^6 x \; dx =$

a.	$\frac{3}{256}\pi$	
b.	$\frac{3}{128}\pi$	
c.	$\frac{3}{512}\pi$	
d	$\frac{5}{256}\pi$	

22. Evaluate $\iiint_R u^2 v^2 w \, du \, dv \, dw$, where R is the region $u^2 + v^2 \le 1$, $0 \le w \le 1$.

a.	$\frac{\pi}{24}$	
b.	$\frac{\pi}{48}$	
c.	$\frac{\pi}{72}$	
d	none of these	

23. The necessary condition for an admissible function to have an extremum of $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')dx$ are

a.	$y(x_1) = y_1, y(x_2) = y_2$	
b.	$y'(x)$ must be continuous	
c.	$y''(x)$ must be continuous	
d	All of these	

24. In Simpson's 1/3rd rule, we replace the graph of the given function by some

a.	Second degree polynomials	
b.	Third degree polynomials	
c.	Fourth degree polynomials	
d	Fifth degree polynomials	

25. If a random variable X follows normal distribution with mean μ and variance α^2 then the random variable $Z = \frac{X-\mu}{\alpha}$ follows normal distribution with

a.	Mean = 1, variance = 0.	
b.	Mean = 0, variance = 1	
c.	Mean = μ, variance = $\frac{\alpha^2}{4}$	
d	none of these	

26. R is set of real numbers and Q is set of rational numbers. Let $f: R \to R$ and $g: R \to R$ be continuous and $f(x) = g(x)\ \forall\, x \in Q$. Then

a.	$f(x) = g(x)$ for some $x \in R/Q$.	
b.	$f(x) = g(x)\ \forall\, x \in R$	
c.	$f(x) \neq g(x)$ for some $x \in R/Q$.	
d	$f(x) \neq g(x)\ \forall\, x \in R$	

27. The directional derivative of ϕ=xyz at the point (1,1,1) in the direction of $\hat{\imath}$ is

a.	-1	
b.	0	
c.	1	
d	2	

28. Consider the following two statements:

(I) A complex valued function $f(z) = u(x,y) + iv(x,y)$ is analytic in a region R.

(II) $f(z)$ is such that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ in a region R.

Choose the correct statement:

a.	I and II are equivalent statements.	
b.	I does not imply II.	
c.	II implies I	
d	II is necessary condition for I, but not sufficient in general.	

29. The partial differential equation

$$\frac{5\partial^2 z}{\partial x^2} + \frac{6\partial^2 z}{\partial y^2} = xy$$

is classified as

a.	Elliptic	
b.	Parabolic	
c.	Hyperbolic	
d	None of the above	

30. The binary representation of decimal no. 25 is

a.	100110	
b.	10011	
c.	11001	
d	110010	

31. For a numerically controlled machine, integers need to be stored in a memory location. The minimum number of bits needed for an integer word to represent all integers between 0 and 1024 is

a.	8	
b.	9	
c.	10	
d	11	

32. The series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

is a Maclaurin series for the following function

a.	a. 8	
b.	b. 9	
c.	c. 10	
d	d. 11	

33. A differential equation is considered to be ordinary if it has

a.	One dependent variable	
b.	More than one dependent variable	
c.	One independent variable	
d	More than one independent variable	

34. The form of the exact solution to

$$\frac{2dy}{dx} + 3y = e^{-x}, y(0) = 5$$

is

a.	$Ae^{-1.5x} + Be^{-x}$	
b.	$Ae^{-1.5x} + Bxe^{-x}$	
c.	$Ae^{1.5x} + Be^{-x}$	
d	$Ae^{1.5x} + Bxe^{-x}$	

35. What is the output of following C++ program?

```
#include <iostream>
using namespace std;
int main() {
    int a, b;
    a = 5;
    b = 10;
    cout << (a==b) << endl;
    cout << (a=b) << endl;
    return 0; }
```

a.	5, 10	
b.	0, 10	
c.	10, 0	
d	Program crashes	

36. The number of non-empty subsets of a set X having 10 elements is:

a.	1023	
b.	1024	
c.	1025	
d	1022	

37. The aim of forward elimination steps in the Gauss elimination method is to reduce the coefficient matrix to the following matrix :

a.	Diagonal	
b.	Identity	
c.	Lower triangular	
d	Upper triangular	

38. Which of the following is **NOT** true for normal curve?

a.	It is skewed.	
b.	It is a probability distribution	
c.	Its total area contains 100% of the cases	
d	The mode, median and mean are identical	

39. Which of the following represent highest correlation?

a.	0.5	
b.	+1.1	
c.	+1.0	
d	-1.0	

40. If p and q are the order and degree of differential equation,

$$y\frac{dy}{dx} + x^3\left(\frac{d^2y}{dx}\right)^3 + xy = cosx$$

then:

a.	$p < q$	
b.	$p = q$	
c.	$p > q$	
d	none of these	

41. If $f(z)$ is regular/analytic except at a finite number of poles within a closed contour C and continuous on the boundary C, then

a.	$\int_C f(z)\,dz = 2\pi i \sum R$	
b.	$\int_C f(z)\,dz = 2\pi \sum R$	
c.	$\int_C f(z)\,dz = \pi i \sum R$	
d	none of these	

42. Evaluate the following integral by using residue theorem

$\int_C \frac{1+z}{z(2-z)}\,dz$ where C is the circle $|z| =$ 1

a.	π	
b.	πi	
c.	$2\pi i$	
d	None of these	

43. Find the area of a parallelogram whose adjacent sites are $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - 4\hat{k}$

a.	$2\sqrt{3}$	
b.	$3\sqrt{5}$	
c.	$5\sqrt{6}$	
d	none of these	

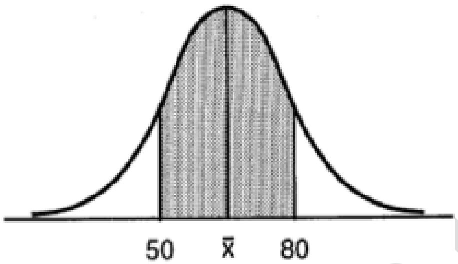
44. Evaluate

$\int_{-\infty}^{\infty} e^{-5t}\delta(t-2)dt$

where $\delta(t-2)$ is the Dirac-delta function.

a.	e	
b.	e^{-10}	
c.	e^{-5}	
d	none of these	

45. In the diagram below, about 68% of the scores fall within the shaded area, which is symmetric about the mean $\bar{x}$. The distribution is normal and the scores in the shaded area range from 50 to 80. What is the standard deviation of the scores in this distribution?



a.	7.5	
b.	15	
c.	30	
d	65	

46. If $^{n}C_{r}$ represents the number of combinations of n items taken r at a time, what is the value of $\sum_{r=1}^{3} {}^{4}C_{r}$

a.	24	
b.	14	
c.	6	
d	4	

47. The number of real solutions of $tan^{-1}\sqrt{x(x+1)} + sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is :

a.	0	
b.	1	
c.	2	
d	∞	

48. Which of the following is correct?

a.	$6 + 3i > 4 + i$	
b.	$3 + i > 4 + i$	
c.	$1 + i > 1 + 5i$	
d	None of the above	

49. The relation { (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)} is

a.	Reflexive	
b.	Symmetric	
c.	Asymmetric	
d	Transitive	

50. The number of leaf nodes in a complete binary tree at depth d is

a.	2^d	
b.	2^d -1	
c.	2^d + 1	
d	2^d + 3	